SEM I

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[5] Theory of Equation

[5.2] Descarte's Rule of signs

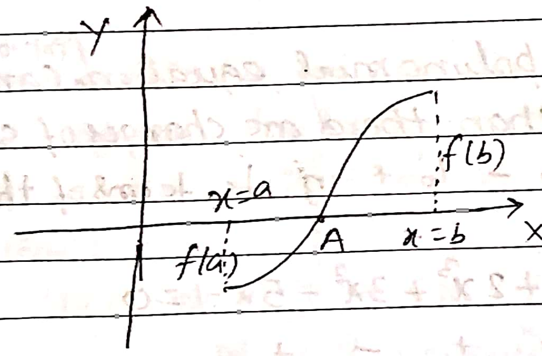
[5.2.1] Introduction:-

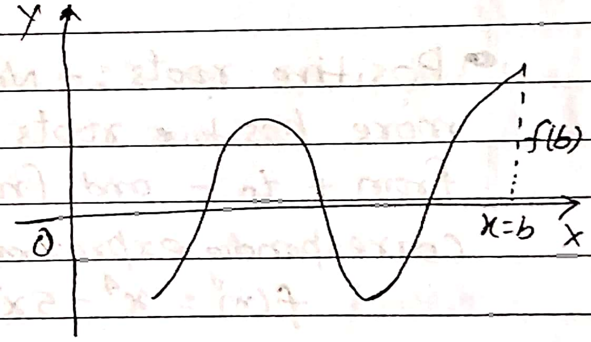
Descarte's rule of sign plays a basic role in the numerical solution of polynomial equations. Descarte's rule of signs enables us to spacify the maximum number of positive and negative roots and consequently the least number of imaginary roots of a given polynomial equation.

[5.2.2] An Important property of Polynomial:-

If two real quantities a and b be substituted for x in any polynomial $f(x)$.

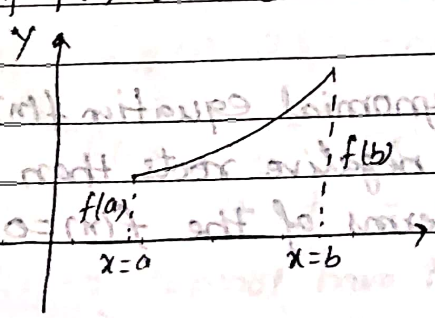
Case I If $f(a)$ and $f(b)$ are ~~of~~ opposite signs $(f(a) \cdot f(b) < 0)$ then, at least one or an odd number of real roots of $f(x) = 0$ lies between a and b.

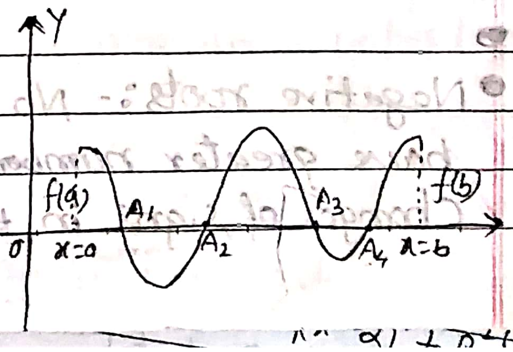




Case II when $f(a)$ and $f(b)$ are of the same sign $(f(a) \cdot f(b) > 0)$

Then, either no real root or an even number of roots of $f(x) = 0$ lie between a and b.





5.2.3 Theorem :- Every equation of an odd degree has at least one real root of a sign opposite to that of the last term.

—DO—

5.2.4 Theorem :- Every equation of an even degree, whose last term is negative has at least two real roots one positive and the other negative.

— DO —

5.2.5 ~~Theorem~~ Descarte's Rule of Signs :-

In the equation $f(x)=0$

(i) The number of positive roots cannot exceed the number of changes of sign in $f(x)$.

(ii) The number of negative roots cannot exceed the number of changes of sign in $f(-x)$.

- Positive roots :- No polynomial equation $f(x)=0$ can have more positive roots than there are changes of signs from + to − and from − to + in its terms of the corresponding expression.

$$f(x) = x^4 - 5x^4 + 2x^3 + 3x^2 - 5x - 1 = 0$$

$$\underbrace{+ \quad -}_{1} \; \underbrace{- \quad +}_{2} \quad + \quad \underbrace{+ \quad -}_{3} \quad +$$

The number of positive roots cannot exceed more than three.

- Negative roots :- No polynomial equation $f(x)=0$ can have greater number of negative roots than there are changes of signs in the terms of the $f(x)=0$.

$f(x) = x^5 - 5x^4 + 2x^3 + 3x^2 - 5x - 1$

$f(-x) = -x^5 - 5x^4 - 2x^3 + 3x^2 + 5x - 1$

$- \quad - \quad \underbrace{- \quad +}_{1} \quad \underbrace{+ \quad -}_{2}$ There are two changes

The number of negative roots cannot exceed than two.

- Complex roots :- Descarte's rule of sign also enables us to find the minimum number of Complex roots of a given polynomial.

Let $f(x) = 0$ be a polynomial of degree n.

Let p = the maximum number of positive roots of $f(x)=0$

q = " " " negative roots of $f(x)=0$

Hence,

The minimum number of Complex roots of $f(x) = 0$ will be $= n-(p+q)$.

- Example① - Using Descarte's rule of sign, find the nature of the roots of the equation

$$x^4 + 15x^2 + 7x - 11 = 0$$

Solution:-

Let $f(x) \equiv x^4 + 15x^2 + 7x - 11 = 0$ —① is of degree 4

It will have four roots, real or Complex.

Again, we have,

$f(x) = x^4 + 15x^2 + 7x - 11 = 0$ have

Signs $+ \quad + \quad + \quad -$ (one change)

$f(x) = 0$ cannot have more than one positive root.

Again,

$f(-x) = x^4 + 15x^2 - 7x - 11 = 0$

$+ \quad + \quad - \quad -$ (one chane)

$f(x) = 0$ cannot have more than one negative root.

$\therefore$ The number of complex roots $= n-(p+q)$
$= 4-(1+1) = 2.$

Example ② Prove that the equation $x^5 - x + 16 = 0$ has two pairs of imaginary roots.

Solution:-

$f(x) = x^5 - x + 16 = 0$ is a polynomial of degree 5

② we have,

$$f(x) = x^5 - x + 16 = 0$$

$+ \quad - \quad +$ (Two changes)

The maximum number of positive roots $= 2$

Again, we have,

$$f(-x) = -x^5 + x + 16 = 0$$

$- \quad + \quad +$ (one change)

The maximum number of negative roots $= 1$

Also,

The minimum number of Complex roots $= 5-(2+1)$
$= 5-3 = 2.$